Gyroscope

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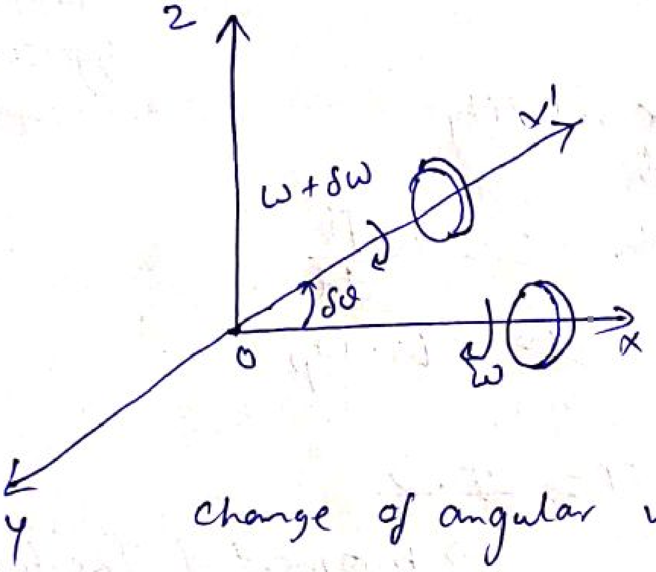
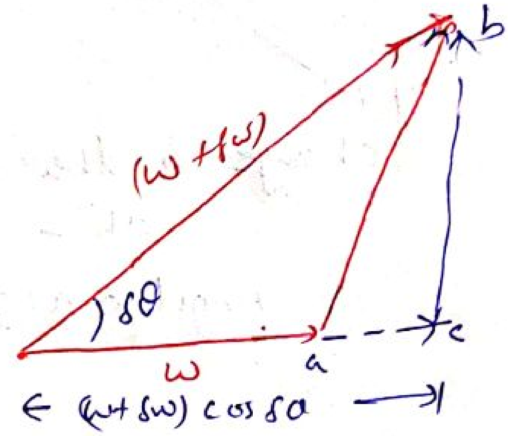
Gyroscope: A groscope is a spinning body which is free to move in other direction under the action of external force.

Precessional Angular Motion:





Change of angular velocity $ac = (\omega+\delta\omega)\cos\delta\theta - \omega$

Angular acceleration $= \lim_{\delta t \to 0} \dfrac{(\omega+\delta\omega)\cos\delta\theta - \omega}{\delta t}$

at $\delta t \to 0$, $\delta\theta \to 0 \to \cos\delta\theta = 1$

$\therefore$ Angular acceleration $\lim_{\delta t \to 0} \dfrac{\omega + \delta\omega - \omega}{\delta t} = \dfrac{d\omega}{dt}$

Change of angular velocity $cb = (\omega+\delta\omega)\sin\delta\theta$

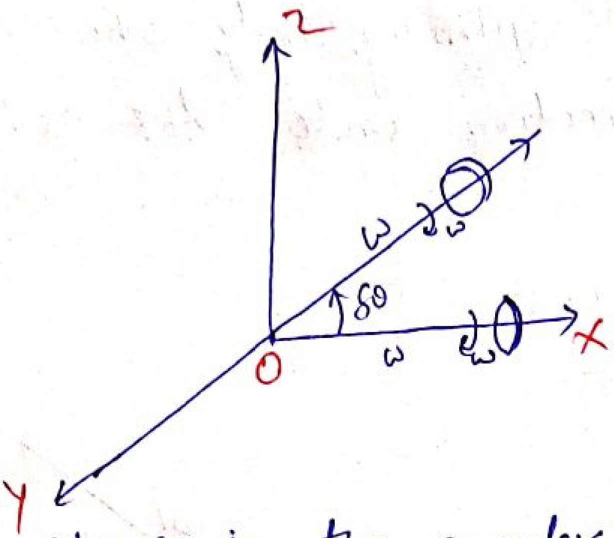
$\therefore$ Angular Acceleration $\lim_{\delta t \to 0} \dfrac{(\omega+\delta\omega)\sin\delta\theta}{\delta t}$

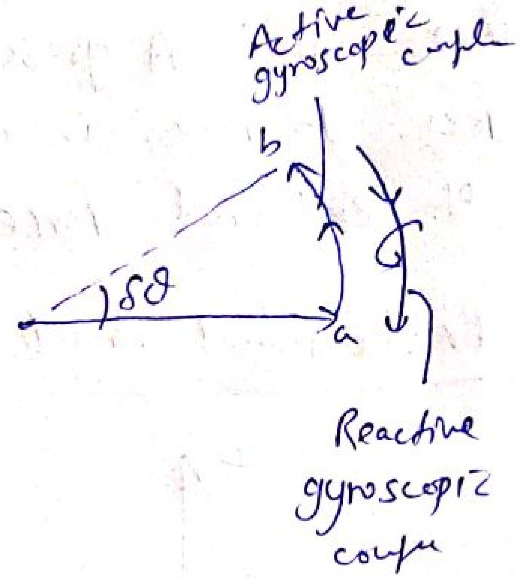
as $\delta t \to 0$ $\delta\theta \to 0 \to \sin\delta\theta = \delta\theta$

$\therefore$ Angular acceleration $= \omega \dfrac{d\theta}{dt}$ → ω_p → precessional velocity

Total Angular acceleration $\alpha = \dfrac{d\omega}{dt} + \omega \cdot \omega_p$

Gyroscopic Couple (Torque):





change in the angular velocity

$$ab = \omega \delta\theta$$

∴ Angular acceleration $\alpha = \omega \frac{d\theta}{dt}$

$$\alpha = \omega \cdot \omega_p$$

Accelerating Torque $T = I \cdot \alpha$

↳ moment of Inertia $= mk^2$

$$T = I \cdot \omega \cdot \omega_p$$

Gyroscopic Couple $C = I \cdot \omega \cdot \omega_p$

- OX → Axis of spin
- OZ → Axis of precession
- OY → Axis of gyroscopic couple
- YZ → plane of spin
- XY → plane of precession
- XZ → plane of gyroscopic couple.

Q1. The turbine rotor of a ship has a mass of 2.2 tonnes and rotates at 1800 rpm, cw when viewed from the left. K = 320 mm, find (i) The ship turns at a radius of 250 m with a speed of 25 km/h (ii) The ship rolls at an angular velocity of 0.1 rad/sec (iii) If it pitches with bow rising at an angular velocity of 0.8 rad/sec

Sol: Given $m = 2200\,kg$, $N = 1800\,rpm$

$R = 250\,m$, $v = 25 \times \frac{5}{18} = 6.94\,m/sec$

$k = 0.32\,m$

$I = mk^2 = 2200 \times (0.32)^2$

$= 225.3\,kg \cdot m^2$

$\omega = \frac{2\pi \times 1800}{60} = 188.5\,rad/s$

$\omega_p = \frac{v}{R} = \frac{6.94}{250} = 0.0278\,rad/sec$

(i) $C = I\omega\omega_p = 225.3 \times 188.5 \times 0.0278 = 1180\,N\text{-}m$ Ans

bow ↓ Stern ↑

(ii) $C = 225.3 \times 188.5 \times 0.1 = 4246.5\,N\text{-}m$ Ans

No effect.

(iii) $\omega_p = 0.8\,rad/sec$

$C = I\omega\omega_p = 225.3 \times 188.5 \times 0.1 = 4246.5\,N\text{-}m$

It will turn the ship towards Right. Ans

Solution of Assignment

(5) Given $m = 8000 kg$, $K = 0.6 m$, $N = 1800$ rpm (CW) from stem

find (i) Gyroscopic couple if $v = 100$ km/h

$= 100 \times \frac{5}{18} = 27.8$ m/sec

$R = 75 m$

$$C = I \omega \omega_p$$

$$= 8000 \times \frac{2\pi \times 1800}{60} \times 100 \times \frac{5}{18} \times \frac{1}{75}$$

$$C = 200.866 \text{ kN-m}$$ **Answer**

(6) Given: Disc dia = 300 mm, mass = 5 kg, $r = \frac{0.3}{2}$ m

$l = 0.6$ $\omega = \frac{2\pi \times 300}{60} = 31.42$ rad/sec

moment of Inertia $I = \frac{m \cdot r^2}{2} = 5 \times \frac{0.15^2}{2} = 0.056 kg m^2$

couple due to mass on the disc

$$C = m \cdot g \times l = 5 \times 9.81 \times 0.6 = 29.43 Nm$$

$$C = I \omega \omega_p$$

$$29.43 = 0.056 \times \frac{2\pi \times 300}{60} \times \omega_p$$

$$\omega_p = 16.7 \text{ rad/sec}$$ **B**

7) Given: flywheel mass = 10 kg, $k = 0.2\,m$

$l = 0.15\,m$ find ω_p

$$\omega = \frac{2\pi \times 900}{60} = 30\pi$$

$$mgl = I\omega\omega_p$$

$$10 \times 9.81 \times 0.15 = 10 \times 0.2^2 \times 3\pi \times \omega_p$$

$$\omega_p = \frac{3.90}{10}\ \text{rad/sec}$$

$$\omega_p = 0.390\ \text{rad/sec}$$